Leptons and quarks in a discrete spacetime

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At the Planck scale I combine a finite subgroup of SO(3,1) operating in 4-dimensional discrete spacetime with a finite subgroup of the Standard Model gauge group $SU(2)_L$ x $U(1)_Y$ x $SU(3)_C$ acting in 4-dimensional discrete internal symmetry space. The unique combination is a specific finite subgroup of SO(9,1) in 10-D discrete spacetime related to E_8 x E_8. The evidence for discreteness would be the appearance of a fourth quark family with its b' quark at about 80-100 GeV decaying to a b quark plus a photon at the Large Hadron Collider.

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I. Introduction

The successful Standard Model (SM) of leptons and quarks describes their electromagnetic, weak and color interactions in terms of symmetries dictated by the $SU(2)_L$ x $U(1)_Y$ x $SU(3)_C$ continuous gauge group. The fundamental particles and antiparticles are defined by their electroweak isospin states in two distinct but gauge equivalent unitary planes in an internal symmetry space 'attached' at a spacetime point. Consequently, particle states and antiparticle states have opposite-signed physical properties but their masses are the same sign, in agreement with empirical data, i.e., there are no negative mass fundamental particles.

The gravitational interaction is not included explicitly in the SM. In an earlier paper (Potter, 1994) I discussed how the SM continuous gauge group is essentially acting in the unitary planes like a "cover group" for the underlying discrete symmetries of its specific finite binary rotational subgroups, thereby suggesting that the internal symmetry space is discrete instead of being continuous. As expected, the mathematical properties of these finite subgroups of the SM were shown to dictate the same physical properties of the leptons and quarks as achieved by the SM . In addition, the finite rotational subgroup approach determined their mass ratios correctly from the j-invariant related to their elliptic modular functions.

Therefore, since mass/energy is the source of the gravitational interaction, there exists the possibility that the gravitational interaction ultimately arises from these discrete symmetries already within the SM. Then not only the internal symmetry space might be discrete but also spacetime itself may be discrete inherently, since gravitation determines the spacetime metric. Spacetime would appear to be continuous only at the low resolution scales of experimental apparatus such as the present particle colliders.

In this paper I assume that my previous assignments of the finite binary rotational subgroups to the lepton and quark families of the SM gauge group will be corroborated at the new Large Hadron Collider within a few years. So I proceed with the next logical step, to mathematically combine a discrete internal symmetry space with a discrete spacetime at the Planck scale of about 10^{-35} meters. The mathematical result is a surprise that unifies the fundamental interactions in a unique way that relates approximately to E_8 x E_8 in superstring theory (also called M-theory).

II. Dimensions of the internal symmetry space?

I take the internal symmetry space of the SM to be discrete, but we need to know how many dimensions there are. Do we need two complex spatial dimensions for a unitary plane as suggested by SU(2), or do we need three as suggested by the SU(3) symmetry of the color interaction, or do we need more?

The lepton and quark particle states are defined as electroweak isospin states by the electroweak part of the SM gauge group, particles in the normal unitary plane C^2 and antiparticles in the conjugate unitary plane C'^2. Photon, W^+, W^-, and Z^0 interactions of the electroweak $SU(2)_L$ x $U(1)_Y$ gauge group operationing in the unitary plane rotate the two particle states (i.e., the two complex basis spinors in the unitary plane) into one another. These electroweak rotations can be considered to occur also in an equivalent 4-dimensional real euclidean space R^4 and in an equivalent quaternion space Q, both these spaces being useful for a better geometrical understanding of the SM.

The quark states are defined also by the color symmetries of $SU(3)_C$, i.e., each quark comes in one of three possible colors, red R, green G, or blue B, while the lepton states have no color charge. Normally, one would consider $SU(3)_C$ operating in a space of three complex dimensions, or its equivalent six real dimensions. In fact, $SU(3)_C$ can operate successfully in the smaller unitary plane C^2, because each SU(3) operation can be written as the product of three specific SU(2) operations (Rowe, Sanders and de Guise, 1999). An alternative geometrical explanation has the gluon operations of the color interaction rotate one color state into another in a 4-dimensional real space, as discussed in my 1994 article. Briefly, real 4-dimensional space R^4 has orthogonal coordinates (w,x,y,z), and its 4-D rotations occur simultaneously in two orthogonal planes. There being only three distinct pairs of orthogonal planes, [wx, yz], [xy, zw], and [yw, xz], each color R, G, or B is assigned to a specific pair, thereby making color an exact geometrical symmetry. Consequently, the gluon operations of $SU(3)_C$ occur in the 4-D real space R^4 that is equivalent to the unitary plane. Detailed matrix operations confirm that hadrons with quark-antiquark pairs, three quarks, or three antiquarks, are colorless combinations.

The internal symmetry space is a discrete 4-dimensional real space because this space allows the SM gauge group to operate completely. One does not need a larger 6-dimensional real space for its internal symmetry space.

III. Dimensions of spacetime?

I take physical spacetime to be 4-dimensional with one time dimension. Spacetime is normally considered to be continuous and 4-dimensional, with three spatial dimensions and one time dimension. However, in the last two decades several approaches toward unifying all fundamental interactions have considered additional mathematical spatial dimensions and/or more time dimensions. For example, superstring theory (Schwarz, 2003) at the high energy regime, i.e., at the Planck scale, proposes 10 or 11 spacetime dimensions in its present mathematical formulation, including the one time dimension. These extra spatial dimensions may correspond to six or seven dimensions ‘curled up’ into an internal symmetry space for defining fundamental particle states at each spacetime point in order to accommodate the SM in the low energy regime. The actual physical spacetime itself may still have three spatial dimensions and one time dimension.

There remains the question of whether spacetime is continuous or discrete. If the internal symmetry space is indeed discrete, then perhaps spacetime itself might be discrete also. Researchers in loop quantum gravity (Urrutia, 2004) at the Planck scale divide spacetime into discrete subunits, considering a discrete 4-D spacetime with its discrete Lorentz transformations to be a viable approach. I therefore take 4-D discrete spacetime as a starting point.

The goal now is to combine the finite subgroups of the gauge group of the SM and the finite group of discrete Lorentz boosts and discrete spacetime rotations into one unified group. All four known fundamental interactions would be unified. Although many unification schemes for the fundamental interactions have been attempted over the past three decades utilizing continuous groups, I believe this attempt is the first one that combines finite groups. Mathematically, the result must be unique, otherwise different fundamental laws could exist in different parts of the universe.

IV. Discrete internal symmetry space

The most important finite symmetry groups in the 4-D discrete internal symmetry space are the 3-D binary rotational subgroups [3,3,2], [4,3,2], and [5,3,2] of the SM gauge group because they contain both 3-D and 4-D discrete rotations and inversions. These are the three subgroups of the SM that I proposed for defining the three lepton families and the lepton states. Being subgroups of $SU(2)_L$ x $U(1)_Y$, they have group operations represented by 2 x 2 unitary matrices or, equivalently, by unit quaternions. Quaternions provide the more obvious geometrical connection (Coxeter, 1974), because quaternions perform the dual role of being a group operation and of being a vector in R^3 and in R^4. One can think visually about the 3-D group rotations and inversions for these three subgroups as quaternions operating on the Platonic solids, with the same quaternions also defining the vertices of regular geometrical objects in R^4.

The two mathematical entities, the unit quaternion q and the SU(2) matrix, are related by

$$q = w + x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \Longleftrightarrow \begin{pmatrix} w+\boldsymbol{i}z & x+\boldsymbol{i}y \\ -x+\boldsymbol{i}y & w-\boldsymbol{i}z \end{pmatrix} \tag{1}$$

where the $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are unit imaginaries, their coefficients are real, and $w^2 + x^2 + y^2 + z^2 = 1$. The conjugate quaternion $q' = w - x\,\boldsymbol{i} - y\,\boldsymbol{j} - z\,\boldsymbol{k}$ and its corresponding matrix would represent the same group operation in the conjugate unitary plane for the antiparticles. Recall that Clifford algebra dictates that only R^4, R^8, and other real spaces R^n with dimensions divisible by four have two equivalent conjugate spaces, the specific mathematical property that accommodates both particle states and antiparticle states. The group $U(1)_Y$ for weak hypercharge Y then reduces the symmetry to being gauge equivalent so that particles and antiparticles have the same positive mass.

One might think that we need to analyze each of the three binary rotational subgroups separately when the discrete internal symmetry space is combined with discrete spacetime. Fortunately, the largest binary rotational group [5,3,2] of icosahedral symmetries can represent the two other groups, and a discussion of its 120 quaternion operations is all inclusive mathematically. The elements of this icosahedral group, rotations and inversions, can be represented by the appropriate unit quaternions in several ways.

The connection between the 3-D and 4-D spaces is realized when one equates the 120 group operations on the regular icosahedron {3, 5} to the vectors for the 120 vertices of the 600-cell hypericosahedron {3, 3, 5} in 4-D space in a particular way. These operations of the binary icosahedral group and the vertices of the hypericosahedron are defined by 120 special unit quaternions q_i known as isosians (Conway and Sloane, 1998), which have the mathematical form

$$q_i = \left(e_1 + e_2\sqrt{5}\right) + \left(e_3 + e_4\sqrt{5}\right)\boldsymbol{i} + \left(e_5 + e_6\sqrt{5}\right)\boldsymbol{j} + \left(e_7 + e_8\sqrt{5}\right)\boldsymbol{k}, \tag{2}$$

where the eight e_j are special rational numbers. Specifically, the 120 icosians are obtained by permutations of

$$(\pm 1,\ 0,\ 0,\ 0),\ (\pm 1/2,\ \pm 1/2,\ \pm 1/2,\ \pm 1/2),\ \text{and}\ (0,\ \pm 1/2,\ \pm g/2,\ \pm G/2), \tag{3}$$

where $g = G^{-1} = G - 1 = (-1+\sqrt{5})/2$. Notice that in each pair, such as $\left(e_3 + e_4\sqrt{5}\right)$, only one of the e_j is nonzero, reminding us that the hypericosahedron is really a 4-D object even though we can now define this object in terms of icosians that are expressed in the much larger R^8 euclidean real space.

So the quaternion's dual role allows us to identify the 120 group operations of the icosahedron with the 120 vertices of the hypericosahedron expressed both in R^4 and in R^8, essentially telescoping from 3-D rotational operations all the way to their representations in an 8-D space. These special 120 icosians are to be considered as special octonions, 8-tuples of rational numbers which, with respect to a particular norm, form part of a special lattice in R^8.

The two other subgroups are next. The 24 quaternions of the binary tetrahedral group [3,3,2] are contained already in the above 120 icosians. So we are left with accommodating the binary octahedral group [4,3,2] into the same icosian format. We need 48 special quaternions for its 48 operations, the 24 quaternions defining the vertices of the 4-D object known as the 24-cell contained already in the hypericosahedron above and another 24 quaternions for the reciprocal 24-cell. The 120 unit quaternions reciprocal to the ones above will meet this requirement as well as define an equivalent set for the reciprocal hypericosahedron, and this second set of 120 octonions also forms part of a special lattice in R^8. Together, these two lattice parts of 120 icosians in each combine to form the 240 octonions of the famous E_8 lattice in R^8, well known for being the densest lattice packing of spheres in 8-D.

Recall that the three binary rotation groups above are assigned to the lepton families because, as subgroups of the SM gauge group, they predict the the correct physical properties of lepton states, including the correct mass ratios. Therefore, the lepton states span only the 3-D real subspace R^3 of the unitary plane. That is why leptons are color neutral and do not participate in the color interaction, which requires the ability to undergo 4-D rotations.

So how do I accommodate the quark states in the icosian picture? The quark states in the SM span the whole 4-D real space, i.e., the whole unitary plane, and I defined them to be the basis states of the 4-D finite binary rotational subgroups of the SM gauge group. But free quarks in spacetime do not exist because they are confined according to QCD, forming the colorless quark-antiquark, three-quark, or three-antiquark combinations called hadrons. Mathematically, these colorless hadron states span the 3-D subspace only, so their resultant discrete symmetry group must be isomorphic to one of the three binary rotational subgroups we have just considered. So the icosians enumerated above account for all the lepton states and for all the quark states as hadronic combinations.

V. Discrete spacetime

Linear transformations in discrete spacetime are described with quaternions for discrete rotations and discrete Lorentz boosts. Before considering these discrete transformations, however, I discuss the continuous transformations of the 'heavenly sphere' as a useful mathematical construct before reducing the symmetry for discrete transformations in a discrete spacetime.

The continuous Lorentz group SO(3,1) contains all the rotations and Lorentz boosts, both continuous and discrete, for the 4-D continuous spacetime with the Minkowski metric. Its operations are quaternions because there exists the isomorphism

$$\mathrm{SO}(3,\ 1) = \mathrm{PSL}\,(2,\ \mathbb{C}). \tag{4}$$

The group $\mathrm{PSL}\,(2, \mathbb{C})$ consists of unit quaternions and is the quotient group $\mathrm{SL}(2, \mathbb{C})/\mathrm{Z}$ formed by its center Z, those elements of $\mathrm{SL}(2, \mathbb{C})$ which commute with all the rest of the group. Its 2 x 2 matrix representation has complex numbers as entries.

The continuous Lorentz transformations (as well as the spatial rotations) operate on the 'heavenly sphere'(Penrose and Rindler, 1987), i.e., the famous Riemann sphere formed by augmenting the complex plane C by the'point at infinity'. The Riemann sphere is also the space of states of a spin-1/2 particle. For the Lorentz transformations in spacetime, if you are located at the center of this 'heavenly sphere' so that the light rays from stars each pass through unique points on a unit celestial sphere surrounding you, then the Lorentz boost is a conformal transformation of the star locations. The constellations will look distorted because the apparent lengths of the lines connecting the stars will change but the angles between these connecting lines will remain the same.

These conformal transformations are called fractional linear transformations, or Möbius transformations, of the Riemann sphere, expressed by the general form (Jones and Singerman, 1987)

$$w \mapsto \frac{\alpha w + \beta}{\gamma w + \delta}\,, \tag{5}$$

with α, β, γ, and δ complex, and $\alpha\delta$-$\beta\gamma \neq 0$. The 2 x 2 matrix representation for transformation of a spinor v as the map $v \mapsto \mathrm{M}v$ is

$$M\ = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}. \tag{6}$$

Thus, M is the spinor representation of the Lorentz transformation. M acts on a vector $\mathrm{A} = \mathrm{vv}^{\dagger}$ via $\mathrm{A} \mapsto \mathrm{MAM}^{\dagger}$ (Manogue and Dray, 1993). All these reltionships are tied together by the group isomorphisms in continuous 4-D spacetime

$$\mathrm{SO}(3,\ 1) = \text{ Möbius group } = \mathrm{PSL}\,(2,\ \mathbb{C}). \tag{7}$$

Discrete spacetime has discrete Lorentz transformations, not continuous ones. These discrete rotations and discrete Lorentz transformations are contained already in SO(3,1), and they tesselate the Riemann sphere. That is, they form regular polygons on its surface that correspond to the discrete symmetries of the binary tetrahedral, binary octahedral, and binary icosahedral rotation groups [3,3,2], [4,3,2], and [5,3,2], the same groups I used in the internal symmetry space for the discrete symmetries. Therefore, the 240 quaternions defined previously are required also for the discrete rotations and discrete Lorentz transformations in the discrete 4-D spacetime. Again, there are the same 240 icosian connections to octonions in R^8 to form a second E_8 lattice.

Thus, the Lorentz group SO(3,1) with its linear transformations in a continuous 4-D spacetime, when reduced to its discrete transformations in a 4-D discrete spacetime, is connected mathematically by icosians to the E_8 lattice in R^8, telescoping the transformations from a smaller discrete spacetime to a larger one. Hence all linear transformations for the particles in a 4-D discrete spacetime have become represented by 240 discrete transformations in the 8-D discrete spacetime.

VI. New spacetime

The discrete transformations in the 4-D discrete internal symmetry space and in the 4-D discrete spacetime are each represented by an E_8 lattice in the 8-D space R^8. The finite group of the discrete symmetries of the E_8 lattice is the Weyl group E_8, not to be confused with the continuous exceptional Lie group E_8. Thus, the Weyl E_8 is a finite subgroup of SO(8), the continuous group of all rotations of the unit sphere in R^8 with determinant unity. In this section I show how the two Weyl E_8 groups combine to form a bigger group that operates in a discrete spacetime, and then in the next section I suggest a simple physical model for fundamental particles that would fit the geometry.

I have now two sets of 240 icosians each forming E_8 lattices in R^8, each obeying the symmetry operations of the finite group Weyl E_8. Each finite group of octonions acts as rotations and as vectors in R^8. I identify their direct product as the elements of a discrete subgroup of the continuous group PSL(2,$\mathbb{O}$), where $\mathbb{O}$ represents all the unit octonions. That is, if all the unit octonions in each were present, not just the subset of unit octonions that form the E_8 lattice, their direct product group would be the continuous group of 2 x 2 matrices in which all matrix entries are unit octonions. So the spinors in R^8 are octonions.

The 8-D result is analogous to the 4-D result but different. Recall that in the 4-D case, one has PSL(2, $\mathbb{C}$), the group of 2 x 2 matrices with complex numbers as entries, with PSL(2,$\mathbb{C}$) = SO(3,1), the Lorentz group in 4-D spacetime. Here in 8-D one has a surprise, for the final combined spacetime is bigger, being isomorphic to a 10-dimensional spacetime instead of 8-dimensional spacetime because

$$\mathrm{PSL}(2, \mathbb{O}) = \mathrm{SO}(9, 1), \tag{8}$$

the Lorentz group in 10-D spacetime.

Applied to the discrete case, the combined group is the discrete subgroup

$$\text{discrete}\,\mathrm{PSL}(2, \mathbb{O}) = \text{discrete}\,\mathrm{SO}(9, 1), \tag{9}$$

that is, the discrete Lorentz group in discrete 10-D spacetime. The same results, expressed in terms of the Weyl E_8 groups, is

$$\mathrm{Weyl}\,E_8 \text{ x } \mathrm{Weyl}\,E_8 = \text{'Weyl'}\,\mathrm{SO}(9, 1), \tag{10}$$

where ‘Weyl’ SO(9,1) is defined by the direct product on the left.

Working in reverse, the discrete 10-D spacetime divides into two parts as a 4-D discrete spacetime plus a 4-D discrete internal symmetry space. There are two surprises in this result: (1) combining a discrete 4-D internal symmetry space with a discrete 4-D spacetime creates a discrete 10-D spacetime, not a discrete 8-D spacetime, and (2) a continuous 10-D spacetime, when ‘discretized’, is not required to partition into a 4-D spacetime plus a 6-D ‘curled up’ space as proposed in superstring theory.

VII. Physical model

My 1994 paper proposed that leptons have the symmetries of the 3-D regular polyhedral groups and that quarks have the symmetries of the 4-D regular polytope groups, these groups being subgroups of the SM gauge group. The analysis hinted that the internal symmetry space could be the discrete 4-D space considered above. Now that I have combined this discrete internal symmetry space with a discrete 4-D spacetime to achieve mathematically a discrete 10-D spacetime, the question arises: Are the leptons and quarks 3-D and 4-D objects physically, or are they something else, perhaps 8-D or 10-D objects?

In order to answer this question I need to formulate a reasonable physical model of fundamental particles in this discrete spacetime environment. The simplest mathematical viewpoint is that discrete spacetime is composed of identical entities, call them nodes, that have no *measureable* physical properties until they collectively distort spacetime to form a fundamental particle such as the electron, for example. The collection of nodes and its distortion of the surrounding spacetime exhibit the discrete symmetry of the appropriate finite binary rotation group for the specific particle. For example, the electron family has the discrete symmetry of the binary tetrahedral group and the electron is one of its two possible orthogonal basis states. So the distortion for the collection of nodes called the electron will exhibit the discrete symmetries of its [3,3,2] group as all of its physical properties emerge for this specific collection and did not exist beforehand. Simultaneously, the positron forms in the conjugate space.

One can begin with a regular lattice of nodes in both the normal unitary plane and in its conjugate unitary plane, or consider the equivalent R^4 spaces, and then imagine that a spacetime distortion appears in both to form a particle-antiparticle pair. Mathematically, one begins with an isotropic vector, also called a zero length vector, which is orthogonal to itself, that gets divided into two unit spinors corresponding to the creation of the particle-antiparticle pair. No conservations laws are violated because their quantum numbers are opposite and the sum of the total mass energy plus their total potential energy is zero. The spacetime distortion that is the particle and its 'field' mathematically brings the nodes closer together locally with a corresponding adjustment to the node spacing all the way out to infinite distance, all the while keeping the appropriate discrete rotational symmetry intact. The gravitational interaction associated with this discrete symmetry therefore extends to infinite distance.

This model of particle creation must treat leptons as 3-D objects and quarks as 4-D objects in a discrete 4-D spacetime. There are no isolated quarks, for they immediately form 3-D objects called hadrons. These lepton states and hadron states are described by quaternions of the form $w + x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$, so these 3-D objects 'live' in the three imaginary dimensions, and the 4th dimension can be called time. Therefore, leptons and hadrons each experience the 'passage of time', while indiviual quarks do not have this freedom until they form hadrons.

If this physical model is a reasonable approximation to describing the world of particles, why are superstring researchers working in 10-dimensions or more? Because one desires a single symmetry group that includes both the group of spacetime transformations of particles and the group of internal symmetries for the particle interactions. At the Planck scale, if one has a continuous group, then the smallest dimensional continuous spacetime one can use is 10-D in order to have a viable Lagrangian. Reducing this 10-D spacetime to the low energy regime of the SM in 4-D spacetime, its 10-D spacetime has been postulated to divide into 4-D spacetime plus an additional 6-dimensional 'curled up' space in which to accommodate the SM. But this approach using continuous groups to connect back to the SM has proven difficult, although some significant advances have been achieved.

The analysis presented above for combining the two discrete Weyl E_8 groups shows that the combined group operates in 10-dimensional discrete spacetime with all the group operations being discrete. No separate 'curled up' space is required at the low energy limit corresponding to a distance scale of about 10^{-23} meters or larger. The discreteness at the Planck scale and the 'hidden' discreteness postulated for all larger distance scales is the mathematical feature that permits the direct unique connection through icosians from the high energy world to the familiar low energy world of the SM.

VIII. Further mathematical connections

The mathematical connections of these binary polyhedral groups to number theory, geometry, and algebra are too numerous to list in this short article. In fact, if one were to choose groups in mathematics upon which to construct the symmetries of the universe, one couldn't choose a better set, for "… in a very profound way, the finite groups of symmetries in 3-space 'see' the simple Lie groups (and hence literally Lie theory) in all dimensions." (Kostant, 1984). I provide a brief survey of a few important connections here and will discuss more of them in detail in a future article.

The continuous group PSL(2,$\mathbb{C}$) defines a torus, as does PSL(2,$\mathbb{O}$). In discrete PSL(2,$\mathbb{C}$) and discrete PSL(2,$\mathbb{O}$) for the discrete spacetime cases, there are special symmetry points on each torus corresponding to the elements of the finite binary polyhedral groups. An important mathematical property of the binary polyhedral groups is their connection to elliptic modular functions and the famous j-invariant function, which has integer coefficients in its series expansion related to the largest of the finite simple groups called the Monster.

The binary tetrahedral, octahedral and icosahedral rotation groups are the finite groups of Mobius transformations PSL(2, Z_3), PSL(2, Z_4), and PSL(2, Z_5), respectively, where Z_n denotes integers mod (n). PSL(2, Z_n) is often called the modular group Γ(n). PSL(2, Z_n) = SL(2, Z_n)/{±I}, so these three binary polyhedral groups (along with the cyclic and dihedral groups) are the finite modular subgroups of PSL(2, $\mathbb{C}$) and are also discrete subgroups of PSL(2, $\mathbb{R}$). PSL(2, Z_n) is simple in only three cases: n = 5, 7, 11. And these three cases are the Platonic groups again: A_5 and its subgroup A_4, S_4, and A_5, respectively (Kostant, 1995).

An important mathematical property for physics is that our binary polyhedral groups, the Γ(n), are generated by the two transformations

$$X : \tau \mapsto -1/\tau \qquad Y : \tau \mapsto \tau + 1, \tag{11}$$

with τ being the lattice parameter for the plane associated with forming the tesselations of the toroidal Riemann surface. The j-invariant function j(τ) of elliptic modular functions exhibits this transformation behavior. Consequently, functions describing the physical properties of the fundamental leptons and quarks will have these same transformation properties. So here is where the duality theorems of M-theory, such as the S duality relating the theory at physical coupling g to coupling at 1/g, arise naturally from mathematical properties of the finite binary polyhedral groups.

Octonions and the triality connection for spinors and vectors in R^8 are related to the fundamental interactions. In 8-D, the fundamental matrix representations both for left- and right-handed spinors and for vectors are the same dimension, 8 x 8 (Baez, 2001), leading to many interesting mathematical properties. For example, an electron represented by a left-handed octonionic spinor interacting with a W^+ boson represented by an octonionic vector becomes an electron neutrino, again an octonionic spinor. Geometrically, this interaction looks like three E_8 lattices combining momentarily to form the famous 24-dimensional Leech lattice!

By using a discrete spacetime, Nature has established a universe based upon fundamental mathematics for fundamental physics principles. I expect that all physical constants will be shown to arise from fundamental mathematical relationships, dictating one universe with unique constant values.

IX. Experimental tests

There is no direct test yet devised for discrete spacetime. However, the discrete internal symmetry space approach I introduced in my 1994 article dictates a fourth quark family with a b' quark at about 80 GeV and a t' quark at about 2600 GeV. The production of this b' quark with the detection of its decay to a b quark and a high energy photon seems to be the only attainable empirical test for discreteness. Its appearance in collider decays would be an enormously important event in particle physics, verifying that the internal symmetry space is discrete and strongly suggesting that the "surrounding" spacetime is discrete.

However, the b' quark has remain hidden among the collision debris at Fermilab because its flavor changing neutral current (FCNC) decay channel has a very low probability compared to all the other particle decays in this energy regime. At present, this b’ quark decay may even be confused with the decay of the Higgs boson, should such a particle exist, until all the quantum numbers are established. The t’ quark at around 2600 GeV has too great a mass to have been produced directly at Fermilab.

I expect the production of b’ quarks at the Large Hadron Collider in a few years to be the acid test for discreteness and to verify the close connection of fundamental physics to the mathematical properties of the finite simple groups.

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